A Hybrid SSA-VMD-ARIMA-CNN-LSTM Model for Carbon Price Forecasting: Empirical Insights from China's Fujian Carbon Market

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Keywords: Carbon Price Forecasting, SSA-VMD Decomposition, Hybrid Model

Abstract: Accurate carbon price forecasting is critical for carbon trading markets to function properly and to meet carbon reduction objectives. However, carbon pricing data are highly nonlinear, non-stationary, and complicated, offering considerable problems to typical econometric models. To solve these challenges, this work introduces a new hybrid forecasting model, the SSA-VMD-ARIMA-CNN-LSTM model. The model uses Variational Mode Decomposition (VMD) to break down the original carbon price series into linear and nonlinear components, with an adaptive Sparrow Search Algorithm (SSA) optimizing VMD settings. An empirical analysis using carbon trading data from Fujian Province shows that the proposed model significantly outperforms single machine learning models and traditional forecasting approaches in terms of accuracy and robustness. The Autoregressive Integrated Moving Average (ARIMA) model is used to model the linear component, while Convolutional Neural Networks (CNN), Long Short-Term Memory (LSTM) networks, and hybrid CNN-LSTM models are applied to various nonlinear components based on their characteristics. The findings offer important insights for investors, businesses, and policymakers in carbon markets.

1. Introduction

Climate change has steadily grown to be a significant factor influencing the political, economic, and social stability of the world in recent years. The most important strategy to stop global climate change and lower the frequency of extreme climatic occurrences is to control greenhouse gas emissions.

One crucial step in achieving the "dual carbon" targets and lowering carbon emissions is the establishment of a carbon emission trading market. [1]The "carbon price" is the cost of carbon emission trading rights. The pricing mechanism, which is the central mechanism of the carbon trading market, significantly affects how the market operates. Carbon pricing is crucial for the Chinese government, businesses, and market to analyze the variables affecting carbon pricing and make accurate predictions about them[2]

Carbon prices often fluctuate due to multiple factors such as economic development, policy changes, energy structure transformation, and market sentiment. This high complexity and uncontrollability give carbon prices distinct characteristics of nonlinearity, non-stationarity, and high complexity.[3] From a theoretical perspective, carbon quota prices include both long-term trend components, reflecting the overall direction influenced by macroeconomic development and carbon reduction policies, as well as short-term high-frequency fluctuation components, such as those influenced by market sentiment, energy supply and demand, and unexpected events. Traditional methods struggle to balance both components, often facing limitations in fitting accuracy[4]. Therefore, models with higher complexity and stronger nonlinear feature learning capabilities are required for forecasting. Based on the above background, this paper proposes the SSA-VMD-ARIMA-CNN-LSTM model, with its structure shown in Figure 1.

  DOI: 10.25236/icmmct.2025.004

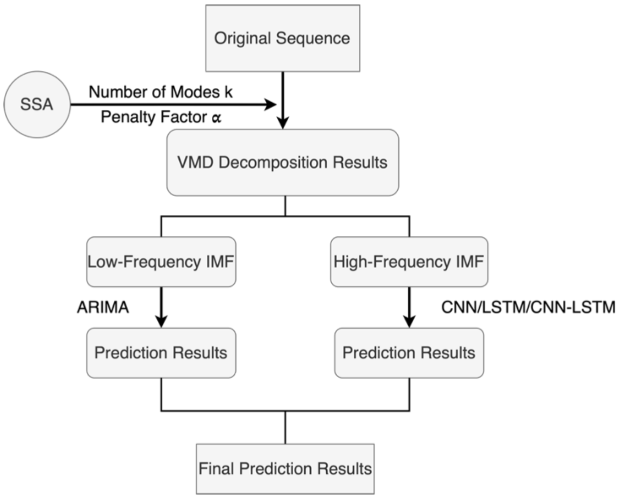


Figure 1 SSA-VMD-CNN-LSTM Model Structure

2. Construction of the SSA-VMD-ARIMA-CNN-LSTM Model and Empirical Analysis

2.1 SSA-VMD Decomposition

Given that the VMD algorithm requires setting the number of modes k and the penalty factor α in advance for practical applications, and that these two parameters significantly affect the final decomposition results[5], This paper uses the SSA to adaptively optimize the parameters, achieving efficient tuning of the VMD decomposition.

2.1.1 SSA Section

The SSA algorithm is inspired by the foraging behavior of sparrow populations, utilizing the roles and interactions between "foragers" and "predator" during the food search process to perform global optimization.[6]This paper applies SSA for the intelligent selection of VMD parameters

① Population Initialization

The position of the sparrow individual

$$\mathrm{X_i} = [\alpha_\mathrm{i}, \mathrm{k_i}] \tag{1}$$

Where α_i and k_i represent the penalty factor and number of modes in VMD. The fitness function is the sum of the permutation entropy (PE) of each IMF after VMD decomposition, used to measure the complexity of the sequence.

② Position Update

The forager position update equation is as follows:

$$X_{ij}^{t+1} = \begin{cases} X_{ij}^{t} \cdot e^{\frac{-i}{\alpha \cdot T}}, R_2 < S_T \\ X_{ij}^{t} + Q \cdot L, R_2 \geq S_T \end{cases} \tag{2}$$

The Predator position update equation is as follows:

$$X_{ij}^{t+1} = \begin{cases} Q \cdot e^{\frac{X_{worst}^{t} - X_{ij}^{t}}{i^2}}, i > \frac{n}{2} \\ X_{p}^{t+1} + \left| X_{ij}^{t} - X_{p}^{t+1} \right| \cdot A^{+} \cdot L, i \leq \frac{n}{2} \end{cases} \tag{3}$$

The sentinel position update equation is as follows:

$$X_{ij}^{t+1} = \begin{cases} X_{best}^{t} + \beta \cdot \left| X_{ij}^{t} - X_{best}^{t} \right|, f_i > f_g \\ X_{ij}^{t} + \kappa \cdot \left(\frac{\left| X_{ij}^{t} - X_{worst}^{t} \right|}{(f_i - f_w) + \varepsilon} \right), f_i \leq f_g \end{cases} \tag{4}$$

The parameters for SSA are set as shown in Table 1.

Table 1: SSA Algorithm Parameter Settings

Population Size	Number of Iterations	Lower Bond	Upper Bond	ST	PD	SD
5	20	[800,4]	[2000,8]	0.7	0.4	0.4

The SSA algorithm begins to converge after approximately 3 iterations. In the initial stage, the fitness value decreases rapidly; subsequently, as the number of iterations increases, the rate of decrease gradually slows, and the curve eventually stabilizes. The penalty factor converges to 1485, and the number of modes converges to 8.

2.1.2 VMD Section

For the original carbon price sequence $P(t)$,VMD attempts to decompose it into k modes $\{u_i(t)\}_{i=1}^{k}$,with corresponding center frequencies $\{\omega_i\}_{i=1}^{k}$

Through Fourier transform, the carbon price sequence $P(t)$ and its corresponding frequency domain can be expressed as:

$$P(t) = \sum_{k=1}^{K} u_k(t) \tag{5}$$

$$U(\omega) = \sum_{k=1}^{K} U_k(\omega) \tag{6}$$

The Fujian carbon price sequence is decomposed using VMD, resulting in 8 IMF subsequences, as shown in Figure 2.

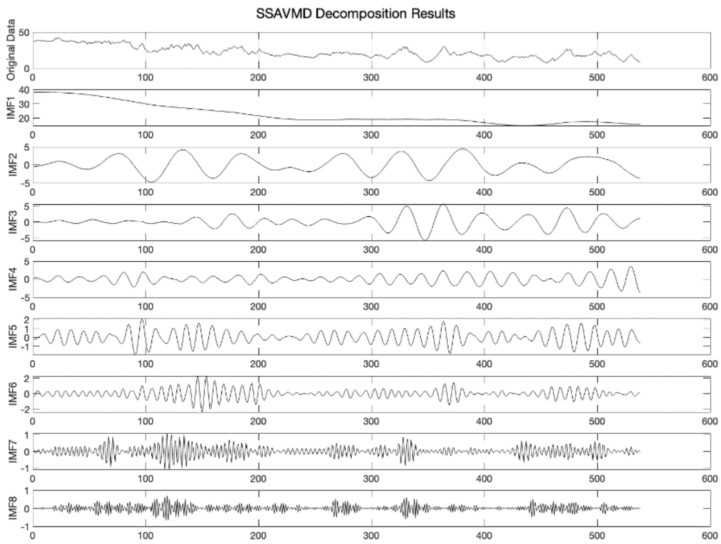


Figure 2 SSA-VMD Decomposition Results

From the figures, it can be observed that the trend of the linear subsequences exhibits a clear long-term direction. In contrast, the nonlinear subsequences do not show an obvious trend but instead display strong random characteristics, oscillating around a certain level.

2.2 Targeted Modeling for Each IMF

2.2.1 Prediction of Linear IMF

The ARIMA model is very good at capturing the linear component of the carbon price series.[7] When an IMF subsequence exhibits relatively stable or linear trend characteristics, the ARIMA(p,d,q) model can be used for prediction. The mathematical form of the ARIMA model can be written as:

$$\Phi_p(B)(1-B)^d IMF_{lin}(t) = \Theta_q(B)\varepsilon_t \tag{7}$$

IMF1 exhibits the long-term trend of the carbon price sequence, and its nonlinearity, non-stationarity, and complexity have been significantly reduced. Therefore, the ARIMA model from traditional statistical models is used to predict IMF1.

The results of the ADF test for IMF1 are shown in Table 2.

Table 2 ADF Test Results for IMF1

Order	Test Statistics	P value
0	-2.956	0.039
1	-2.362	0.153
2	-3.636	0.005

Further analysis of IMF1 was conducted, and the ACF and PACF plots for IMF1 are shown in Figure 3.

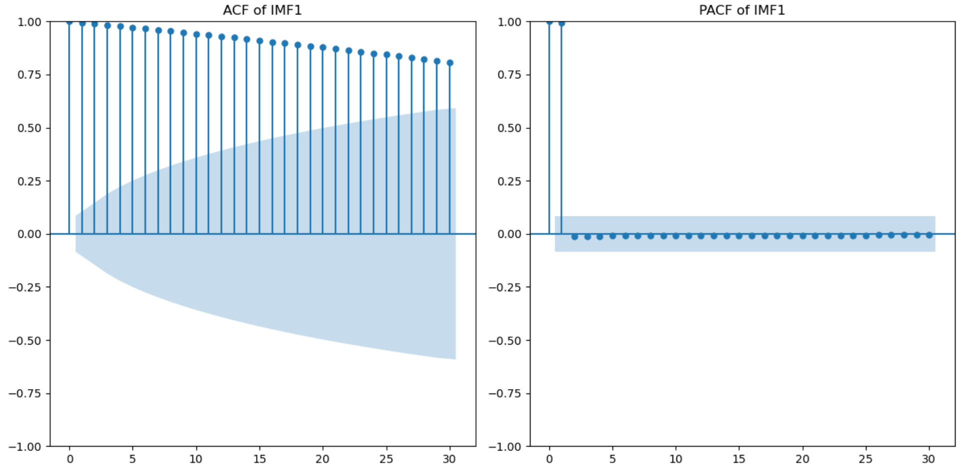


Figure 3 The ACF and PACF plots for IMF1

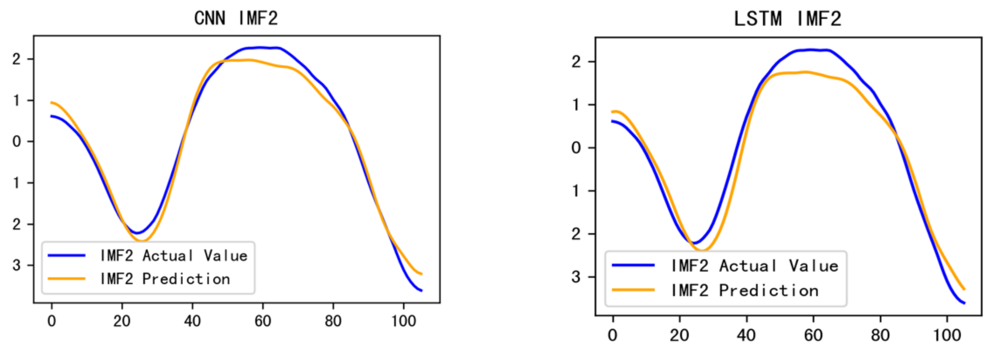
Based on this result, suitable parameters were selected and the ARIMA model was used to fit IMF1, with the results shown in Table 3.

Table 3: ARIMA Fitting Results for IMF1

Evaluation Metrics	R^2	MSE	RMSE	MAE
Value	0.9999	4.9057×10^{-5}	0.0070	0.0055

2.2.2 Prediction of Nonlinear IMF

Given the significant non-stationarity and complexity of the nonlinear subsequences, complex models with strong feature learning capabilities are needed for fitting. This paper uses single CNN, single LSTM, and CNN-LSTM models to fit and predict IMF2-IMF8, selecting the best model as the final result. [8]The prediction results are shown in Figure 7, with the results for IMF2 presented as an example.



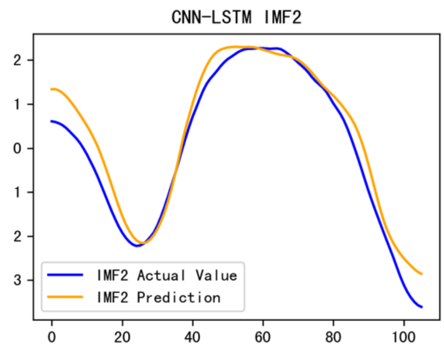


Figure 4 The Prediction Result of IMF2

The results show that for IMF2, the prediction performance of the single LSTM model is the worst. To compare the prediction accuracy more specifically, a summary of the prediction accuracy for different models is provided in Table 4.

Table 4 IMF2 Prediction Accuracy Summary Table

Model	MSE	RMSE	MAE	R^2
LSTM	0.0053	0.0728	0.0598	0.8548
CNN	0.0010	0.0310	0.0265	0.9736
CNN-LSTM	0.0018	0.0422	0.0356	0.9512

The final chosen prediction models for each IMF component are shown in Table 5.

Table 5: Prediction Models for Each Component

Component	IMF1	IMF2	IMF3	IMF4	IMF5	IMF6	IMF7	IMF8
model	ARIMA	CNN	CNN-LSTM	CNN	LSTM	LSTM	CNN	CNN

3. Model Evaluation

3.1 Evaluation of VMD Algorithm Effectiveness

Strategy 1: Directly use CNN-LSTM to predict the overall carbon price sequence

Strategy 2: After applying VMD decomposition, use CNN-LSTM to predict each subsequence, and then combine them to predict the overall sequence.

By comparing Strategy 1 and Strategy 2, the effect of VMD decomposition on carbon price sequence prediction can be determined. The comparison is shown in Figure 5.

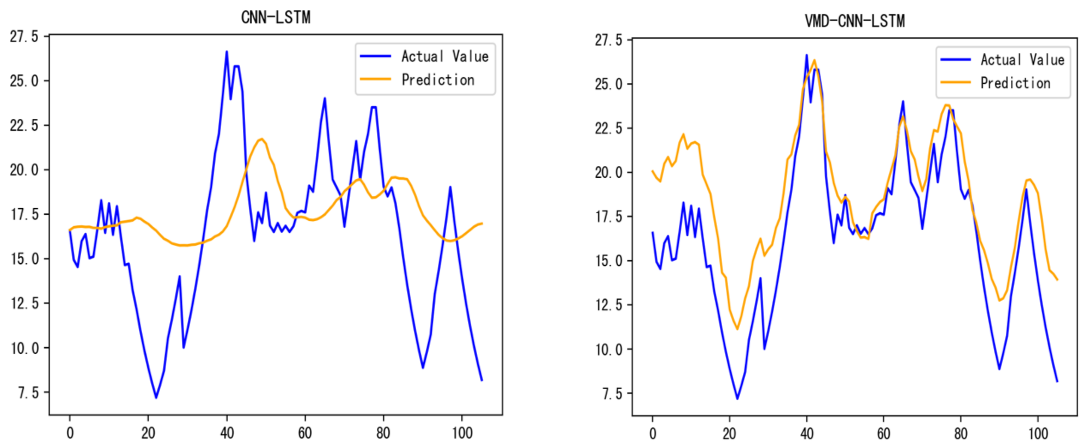


Figure 5 Comparison of Prediction Effect with and without VMD

The comparison of prediction accuracy with and without the VMD algorithm is shown in Table 6.

Table 6 Comparison of Prediction Accuracy with and without VMD

	MSE	RMSE	MAE
CNN-LSTM	18.6322	4.3165	3.3928
VMD-CNN-LSTM	7.1316	2.6705	2.1911

3.2 Evaluation of SSA for VMD Parameter Optimization

As mentioned earlier, the parameters for VMD are limited to a number of modes between 4 and 8, and a penalty factor between 800 and 2000. The comparison between manually selected parameters and those optimized by SSA for the model parameters demonstrates the effect of SSA on VMD parameter optimization. The comparison results are shown in Table 7.

Table 7 Comparison of SSA Parameter Optimization and Manual Parameter Selection

Parameter(k,$\boldsymbol{\alpha}$)	MSE	RMSE	MAE
(5,1000)	11.6846	3.4183	2.8000
(6,1200)	11.0561	3.3250	2.7926
(7,1400)	9.9046	3.1471	2.7000
(8,1898)	7.1316	2.6705	2.1911

3.3 Comparison of Single Prediction Models and Hybrid Prediction Models

Strategy 1: Use CNN for each component prediction
Strategy 2: Use LSTM for each component prediction
Strategy 3: Use CNN-LSTM for each component prediction
Strategy 4: Select the optimal model for each component and construct a hybrid prediction model
A comparison of all strategies is shown in Figure 6 and Table 8.

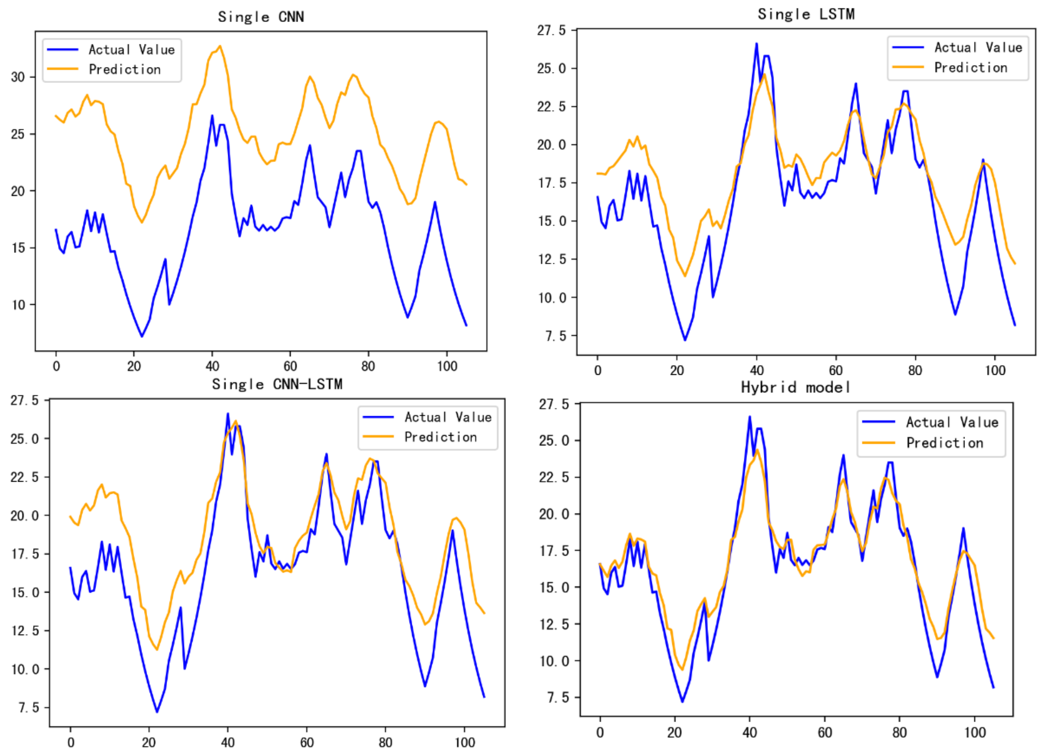


Figure 6 Comparison of Single Prediction Models and Hybrid Prediction Models

Table 8 Comparison of Single and Hybrid Prediction Model

	MSE	RMSE	MAE
Single CNN	58.1090	7.6229	7.4209
Single LSTM	4.6126	2.1477	1.7830
Single CNN-LSTM	7.1316	2.6705	2.1911
Hybrid Model	1.4415	1.2006	0.9795

From Figure 6 and Table 8, it can be seen that selecting the appropriate model for each IMF component, as opposed to using a unified single model, greatly improves the prediction capability of the model.

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